Advanced in Flexible Robots for Surgical Interventions

ICRA 2014 FULL-DAY WORKSHOP

Saturday May 31st 2014
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Contents

1 Introduction

1.1 Organizers

Dr. Christos Bergeles
c.bergeles@imperial.ac.uk

Dr. Su-Lin Lee
su-lin.lee@imperial.ac.uk

Prof. Guang-Zhong Yang
g.z.yang@imperial.ac.uk

Prof. Pierre E. Dupont
pierre.dupont@childrens.harvard.edu

Prof. Koji Ikuta
ikuta@rcast.u-tokyo.ac.jp

Contact:
Dr. Christos Bergeles
The Hamlyn Centre
Imperial College London
Exhibition Road
London SW7 2AZ
07593700258

1.2 Abstract

A broad variety of flexible surgical robots aiming to overcome the limitations of traditional rigid-instrument based laparoscopic surgery are being proposed. These robots can take advantage of their flexible structure, and be introduced through natural orifices, navigate anatomical pathways, and perform localised interventions with an ever increasing accuracy and trauma reduction. The technologies behind these surgical robots span concentric tube based devices, tendon driven mechanics, hydraulically controlled actuators, miniaturised universal joints and many more. With the maturity of the surgical robotics field in the recent years, these technologies are coming closer to clinical translation, even though several hurdles must still be overcome. This workshop aims to bring wordwide researchers together in order to explore the common goals and limitations, and exploit the commonalities of the different systems. This full-day workshop will include researchers from academic, industrial, and clinical environments, in order to identify unifying research questions and approaches in the design, implementation, and evaluation of flexible access robots for surgical interventions.

1.3 Statement of Objectives

Existing minimally invasive surgical robots based on rigid links have improved surgical outcomes and reduced patient trauma by increasing surgical dexterity and providing depth-perception capabilities to the surgeon. As this technology matures, researchers are setting more demanding goals: further miniaturisation of surgical robotic platforms, reduction of trauma by minimising entry incisions to a single one, and further increase of dexterity to navigate and operate through tortuous natural pathways. These new procedures are referred to as flexible access endoluminal and transluminal interventions.

Flexible access robots are a technology promising to revolutionise both endoluminal and transluminal surgeries. Developed platforms comprise miniaturised mechatronic components to create arm-like assemblies with active shape control, and their flexibility allows insertion through natural orifices and operation inside anatomical lumen, e.g., the stomach or the abdomen. The field of flexible surgical robots is gaining great momentum, and several research groups and companies are demonstrating impressive achievements. Nonetheless, there are several important technical problems that still need to be addressed.
Optimal design of robots and accurate mechanics and dynamics modelling are the basis for operational accuracy. Moreover, the mechanical structures of the robots should exhibit varying and controlled flexibility, ideally being able to adjust between rigid and flexible configurations. Procedure planning and real-time control are additional challenges that need to be addressed, as are the force sensing, visualisation capabilities, and manufacturing limitations of ever smaller robots and miniaturised tools. Developing any of these robotics systems and medical devices for clinical use shares this common set of problems. Additionally, such interventional devices share common challenges with respect to clinical acceptance.
Despite these interconnections, however, it has been several years since a dedicated event. This workshop will bring researchers together to identify the unifying themes and solution strategies for this class of medical robots, to build new partnerships, and to spark new ideas for moving the field forward.

1.4 Intended Audience

The primary audience of the workshop consists of researchers and their students from academia, industry and clinical practice who are currently investigating flexible surgical robots. The secondary audience consists of those researchers who are interested in applying this class of robots to new medical applications, and those investigators with a broader interest in actuators and mechanisms for flexible robots.

1.5 List of Topics

- Snake-like robots
- Bimanual dextrous flexible robotic platforms
- Reconfigurable endoluminal robots
- Steerable catheters
- Concentric tube robots
- Steerable needles

2 Program

2.1 Invited Talks

Each talk will last 25 minutes and 5 additional minutes will be allocated for questions and speaker change. During coffee breaks and lunch break, the audience will have the chance to view the posters.

2.2 Poster Teasers

Each poster teasers will last 3 minutes including speaker change. Poster viewing will occur during the lunch break and coffee breaks.

Session 1

09:00am - 09:30am	G.-Z. Yang	Challenges and Opportunities in Flexible Access Surgical Robots
09:30am - 10:00am	Y.-J. Kim, J. Lee	A Novel Trans-Umbilical Single-Port Surgical Robot Maximizing Internal Dexterity, Flexibility and Stiffness
10:00am - 10:30am	N. Simaan	Continuum Robots for Single Port Access, Transurethral, Throat and Cochlear Implant Surgery
10:30am - 10:50am	Coffee, Posters	

Session 2

10:50am - 11:20am	P. Dupont	Exteroceptive and Proprioceptive Sensing for Co-robotic Control of Surgical Continuum Robots
11:20am - 11:50pm	R. Webster	Transurethral, Transbronchi, and Transtissue: New Flexible Access Surgeries Enabled by Concentric Tube Robots
11:50am - 12:20pm	J. Desai	Towards Robot-Assisted Neurosurgery Under Continuous MRI Guidance
12:22pm - 12:40pm	Poster Teasers	
12:40pm - 14:00pm	Lunch Break	

Session 3

14:00pm - 14:30pm	P. Chiu	Clinical Challenges in Flexible Access Surgery
14:30pm - 15:00pm	K. Althoefer	From STIFF to FLOPpy: A New Approach for Robot-Assisted Surgery: Advancements and Challenges
15:00pm - 15:30pm	A. Menciassi	Endoluminal and Endocavitary Robots: Some Examples for Combining Flexibility and Dexterity
15:30pm - 16:00pm	Coffee, Posters	

Session 4

16:00pm - 16:30pm	K. Ikuta	From Flexible Endoscopes to Nanotools for Single-Cell Surgery
16:30pm - 17:00pm	D. Camarillo	Model-less Control for Continuum Manipulators in Unstructured Environments
17:00pm - 17:30pm	R. Patel	Control of Tip Contact Force for Steerable Ablation Catheters
17:30pm - 18:00pm	B. Nelson	Magnetic Navigation of Flexible Access Robots
18:00pm - 18:20pm	Closing	

Poster Teaser and Poster Presentation

12:22pm - 12:25pm	A Novel Underactuated Wire-Driven Flexible Robotic Arm with Controllable Bending Section Length
12:25pm - 12:28pm	Teleoperated Robotic Needle Steering with Cartesian Space Control
12:28pm - 12:31pm	Model-less Control of a Flexible Robotic Catheter
12:31pm - 12:34pm	Automated Pointing of Cardiac Ultrasound Catheters
12:34pm - 12:37pm	Nonlinear Information Fusion for Snakelike-Robot Backbone-Frame Localization
12:37pm - 12:40pm	Bifurcated Paths for Steerable Flexible Needles

3 Invited Speakers

3.1 Kaspar Althoefer

Affiliation

Centre for Robotics Research, Department of Informatics, King's College, London, UK

Talk Title

From STIFF to FLOPpy: A new approach for robot-assisted surgery: Advancements and Challenges

Abstract

The last decade has seen tremendous technological advancements in the field of Robot-assisted Minimally Invasive Surgery (RMIS). Robotic surgical systems, such as the da Vinci system by Intuitive Surgical have penetrated the operating theatre and have shown to represent a suitable alternative to laparoscopic surgery, at least for a number of procedures such as prostatectomy. Its main advantage over existing techniques is that it allows surgeons to conduct complex procedures in an intuitive way while providing 3D views of the operating area. Limitations, though, stem from the fact that such manipulation devices are built from straight, rigid links and lack tactile sensing modalities as well as haptic feedback. More recent research efforts have focussed on creating surgical robots whose structure is flexible allowing the robot to follow more complex trajectories without negatively impacting on healthy tissue, including systems such as the i-snake (Imperial College) and HARP (Carnegie Mellon University) and concentric tube robots (Webster/Dupont). Departing from these types of robots, which are fundamentally based on a structure made from rigid link elements, EU project STIFF-FLOP proposes a new concept of modern, inherently safe robots for minimally invasive surgery, capable of morphing from a stiff to a soft state. Inspired by the octopus, the vision of the project is to develop a fully-integrated surgical robot system, combining soft and stiffness-controllable mechanisms, pneumatic and hydraulic actuation, tactile and force sensors, haptics as well as advanced control and learnable navigation techniques. The presentation will give an overview of the STIFF-FLOP project, the advancements to date and the challenges that lie ahead.

3.2 David Camarillo

Affiliation

Camarillo Lab, Stanford, USA

Talk Title

Model-less control for continuum manipulators in unstructured environments

Abstract

Continuum manipulators are capable of navigating through unstructured environments such as a human body, where its flexible backbone allows it to safely conform around environment obstacles and constraints. Because these environmental disturbances can affect the configuration of the manipulator in an unknown and unpredictable manner, conventional kinematic and mechanics model-based control methods can be inaccurate and can lead to robot singularities and instabilities. We have developed a novel control method that uses local Jacobian estimation and convex optimization to control a continuum manipulator without using a model. We show that this method overcomes artificial singularities and instabilities that may otherwise occur using a model-based controller, and can be used in a variety of environmental

constraints and obstacles without explicit knowledge of the environment. Model-less control provides a robust control strategy for continuum manipulators in unstructured environments, and offers a simple and effective alternative for controlling robot manipulators with complex kinematics and mechanics.

3.3 Jaydev Desai

Affilation

Robotics Automation and Medical Systems (RAMS) Laboratory, Department of Mechanical Engineering, University of Maryland, USA

Talk Title

Towards Robot-Assisted Neurosurgery Under Continuous MRI Guidance

Abstract

Brain tumors are among the most feared complications of cancer occurring in 20%-40% of adult cancer patients. Though there have been significant advances in treatment, the prognosis for these patients is poor. Whether there is a primary malignancy or a secondary malignancy, whenever the brain of the cancer patient is involved in treatment, there is a significant impact on their overall quality of life. While the most optimal treatment currently for most brain tumors involves primary surgical resection, many patients may not be able to undergo that treatment plan due to either their poor general health or an unfavorable location (either deep inside the brain or inaccessibility of the tumor) of the lesion. Magnetic resonance imaging (MRI) provides excellent soft tissue contrast and has become a standard imaging modality for physicians in several image-guided interventions. However, the nature of MR imaging imposes several constraints on the development of a robotic system. These challenges include actuator choice, sensor choice, material choice, size of the robot, etc., to name a few. This talk will focus on our progress on the development of MINIR: Minimally Invasive Neurosurgical Intracranial Robot, and identify the challenges in the development of this meso-scale robotic system operated under MRI guidance.

3.4 Pierre E. Dupont

Affiliation

Pediatric Cardiac Bioengineering Lab, Boston Children's Hospital, Harvard Medical School, Boston, USA

Talk Title

Exteroceptive and Proprioceptive Sensing for Co-robotic Control of Surgical Continuum Robots

Abstract

Continuum robots provide the potential to perform interventions with substantially less trauma compared to current robotic technologies employing straight rigid instrument shafts. The 3D curvature and flexibility of continuum robots that bestow this advantage, however, substantially complicate the clinician?s task of controlling robot-tissue interaction not only at the robot's tip, but along its entire length. Co-robotic controllers that automatically adjust tissue contact forces and compensate for robot deflection can improve safety and enhance performance while enabling the clinician to focus on the task being performed at the robot's tip. A major challenge, though, is that sensing technology has not kept pace with developments in continuum robot design. To address this, we have been exploring concepts for creating exteroceptive sensor arrays to measure tissue contact, pressure or force. These are embodied as robot fingertips and as thin elastic skins covering the robot's surface. We have also been exploring

proprioceptive sensors to measure 3D robot shape. In this talk, I will share our clinical motivations for this work, describe our progress and outline the challenges ahead.

3.5 Koji Ikuta

Affiliation

University of Tokyo, Japan

Talk Title

From Flexible Endoscopes to Nanotools for Single-Cell Surgery

3.6 Yong-Jae Kim and Jongwon Lee

Authors and Affiliations

Yong-Jae Kim
Korea University of Technology and Education, Cheonan, Korea

Jongwon Lee
Samsung Advanced Institute of Technology

Talk Title

A Novel Trans-Umbilical Single-Port Surgical Robot maximizing Internal Dexterity, Flexibility and Stiffness

Abstract

A new surgical robotic system for a trans-umbilical single-port access surgery is presented, which has high internal dexterity and high stiffness in a compact size. It includes a snake-like guide tube, two 7-DOF surgical tools, and one 3-DOF stereo endoscope. To overcome the critical limitations of conventional single-port surgical robots, concerning with internal dexterity and stiffness, two novel mechanisms are proposed: 1) A unique snake-like guide tube based on a variable-stiffness mechanism enabling access of the entire abdominal cavity in arbitrary angles of approach as well as controlling the stiffness for operational safety. 2) Multi-DOF surgical tools based on a wire reduction mechanism amplifying the tension of joint actuating wires that provides sufficient stiffness of the surgical tools and endoscope. The developed surgical robot based on these mechanisms was verified by simulations and experiments.

3.7 Arianna Menciassi

Affiliation

The BioRobotics Institute, Scuola Superiore Sant'Anna, Pisa, Italy

Talk Title

Endoluminal and endocavitary robots: some examples for combining flexibility and dexterity

Abstract

This talk will focus on the design of endoluminal and endocavitary robots with enhanced flexibility, which is obtained by combining soft materials with fluidic actuation. Starting from previous developments of bioinspired inchworm locomotion robots for colonoscopy, the speaker will present the current approach to develop surgical manipulators without rigid links and with controllable stiffness. Thus, an analysis about the difficulties of combining flexibility with dexterity will be presented. A reconfigurable endocavitary platform will be finally presented: in this case, reconfiguration can answer the need of flexibility, by changing the approach from soft and controllable materials to miniature assembling bricks for building multiple and variegated surgical robots.

3.8 Bradley J. Nelson

Affiliation

Institute of Robotics and Intelligent Systems, ETH Zurich, Switzerland

Talk Title

Magnetic navigation of flexible access robots

Abstract

We are developing a variety of technologies based on microrobotics for applications ranging from targeted therapies for the retina, for individual cells, and for cardiac ablation, as well as micromanipulation for industrial automation. A key aspect of the approaches entails the use of externally generated magnetic fields and field gradients to precisely control the motion of these devices. As we move towards the nanoscale, motion strategies inspired by bacterial motors are used.
The futuristic vision of micro and nanorobotics is of intelligent machines that navigate throughout our bodies searching for and destroying disease, but we have a long way to go to get there. Progress is being made, though, and the past decade has seen impressive advances in the fabrication, powering, and control of tiny motile devices. Much of our work focuses on creating systems for controlling micro and nanorobots in liquid as well as pursuing applications of these devices. Larger scale microrobots for delivering drugs to the retina to treat eye diseases such as age related macular degeneration and retinal vein and artery occlusion are moving towards clinical trials. As size decreases to the nanoscale, we have been inspired by motile bacteria, such as E. coli, and have developed nanorobots that swim with a similar technique. Applications we pursue at these scales are for the treatment of breast cancer and cerebral infarctions. The potential impact of this technology on society is high, particularly for biomedical applications, though many challenges remain in developing micro and nano robots that will be useful to society. An overarching requirement for achieving breakthroughs in this area is the need to bring together expertise from a wide variety of science and engineering disciplines. Robotics brings expertise in the planning and control of mechanisms with many degrees of freedom in uncertain environments. Nanotechnology teaches innovative approaches to fabricating nanoscale machines. In addition, biomedical imaging advances are needed, as is fundamental insight into the nature of fluid dynamics at very small scales. Medical professionals must be tightly integrated into the development cycle, and experts in developing business models and intellectual property must be closely consulted.
As systems such as these enter clinical trials, and as commercial applications of this new technology are realized, radically new therapies and uses will result that have yet to be envisioned.

3.9 Rajni Patel

Affiliation

Department of Electrical and Computer Engineering, University of Western Ontario, Canada

Talk Title

Control of Tip Contact Force for Steerable Ablation Catheters

Abstract

Catheter-based cardiac ablation is a well-accepted treatment for atrial fibrillation, a common type of cardiac arrhythmia. During this procedure, a steerable ablation catheter is guided through the vasculature to the left atrium. The catheter is then used to ablate some parts of cardiac tissue to restore normal heart rhythm. An important factor that affects the efficacy of the ablation procedure is the contact force between the catheter tip and cardiac tissue and it is desired to maintain a constant contact force in spite of cardiac motion. However, in the conventional method of performing cardiac ablation, force information is not available and alternate techniques for estimating the contact force need to be developed. In the context of continuum robots, estimating contact forces using information from sources other than force sensors has been a topic of recent interest. This talk will focus on the design of a robotics-assisted catheter manipulation system with contact force control for use in cardiac ablation procedures. As a basic step in this regard, the behavior of current ablation catheters in different scenarios will be discussed to better understand their features and limitations. In the static case, it will be shown that it is possible to determine the tip/tissue contact force from the catheter shape without installing a force sensor on the catheter. During cardiac ablation, the catheter tip is in contact with a relatively fast-moving environment (cardiac tissue). For maintaining a constant contact force, the catheter tip should be actuated at frequencies close to those of cardiac motion. Robotic manipulation of the catheter has the potential to provide such motion. The talk will present some recent results on different properties of ablation catheters, e.g., how the catheter tip deflects when the catheter handle is actuated and when the catheter comes in contact with static as well as moving environments. Techniques for controlling the contact force and their limitations will be discussed. Based on these results, a modified actuation technique as well as suggestions for improved catheter design will be presented.

3.10 Nabil Simaan

Affiliation

ARMA Lab, University of Vanderbilt, USA

Talk Title

Continuum Robots for Single Port Access,
Transurethral, Throat and Cochlear Implant Surgery

3.11 Robert Webster

Affiliation

Medical and Electromechanical Design Lab, University of Vanderbilt, USA

Talk Title

Transurethral, Transbronchi, and Transtissue: New Flexible Access Surgeries Enabled by Concentric Tube Robots

Abstract

Concentric tube robots are one of the thinnest flexible surgical robots available, which makes them a powerful tool in the armamentarium of surgeons and engineers as they partner to combat challenging diseases. In addition to the transnasal and transvascular approaches that have been most well studied to date, concentric tube robots are poised to enable novel procedures in the prostate, lung, and brain, among other organs. This talk will review our recent progress on creating multi-arm systems that can be delivered through endoscope ports, forming tube sets that can act like steerable needles, and creating image-guided surgery systems that make use of the unique capabilities of concentric tube robots.

3.12 Guang-Zhong Yang

Affiliation

The Hamlyn Centre, Imperial College London, UK

Talk Title

Challenges and Opportunities in Flexible Access Surgical Robots

4 Short Papers Abstracts

4.1 A Novel Underactuated Wire-Driven Flexible Robotic Arm with Controllable Bending Section Length

Authors

Zheng Li, Haoyong Yu, and Hongliang Ren

Affiliation

National University of Singapore, Singapore.

Abstract

This paper presents a novel underactuated wire-driven flexible robotic arm with controllable bending section length. It comprises an underactuated wire-driven flexible robotic arm, a rigid translating tube and the tube holder. The bending motion of the robotic arm is constrained by the tube. By translating the tube, the bending section length is controlled. With this design, the end-effector is able to reach more positions, or the workspace is increased. This is beneficial for surgical interventions or other operations in confined space.

4.2 Teleoperated Robotic Needle Steering with Cartesian Space Control

Authors

Ann Majewicz, and Alison Okamura

Affiliations

Department of Mechanical Engineering, Stanford University, Stanford, CA, USA.

Abstract

Asymmetric-tipped, robotically controlled steerable needles have the potential to improve clinical outcomes for many needle-based procedures by allowing the needle to curve and change direction within biological tissue. These needles are nonholonomic systems, which can be difficult to control manually using joint-space inputs of insertion and rotation at the needle base due to unintuitive kinematic constraints. A new method of controlling steerable needles through teleoperation of the needle tip in Cartesian space has been shown, in simulation, to result in significantly less targeting error and time to reach the intended target. In this work, we describe methods to implement Cartesian space teleoperation with a robotic needle steering system and present preliminary evaluation results.

4.3 Model-less Control of a Flexible Robotic Catheter

Authors

Michael Yip, David Camarillo

Affiliation

Department of Bioengineering, Stanford University, Stanford, CA, USA.

Abstract

Flexible robotics are capable of navigating through unstructured environments such as a human body, where it can safely conform around environmental obstacles and constraints due to its compliant body. Because these environmental disturbances can affect the configuration of the robot in an unknown and unpredictable manner, conventional kinematics- and mechanics-based control methods can be inaccurate and can lead to robot singularities and controller instabilities. Using model-less control, we demonstrate control of a flexible robot manipulator without requiring a model. Furthermore, we show its application in a tissue ablation task using a flexible catheter. Model-less control provides a robust control strategy for flexible manipulators in unstructured environments, and offers a simple and effective alternative for controlling robots with complex kinematics and mechanics.

4.4 Automated Pointing of Cardiac Ultrasound Catheters

Authors

Paul Loschak, Laura Brattain, and Robert Howe

Affiliation

Biorobotics Lab, Harvard University, Boston, MA, USA.

Abstract

Automatically positioning cardiac imaging catheters within the heart can improve diagnoses and treatments of medical conditions such as cardiac arrhythmias, including atrial fibrillation. Imaging catheters are unique compared to many position-controlled flexible instruments because device orientation must be specified in order to point the imager at the target. For example, ultrasound imaging (intracardiac echo, or ICE) catheters are steered by four actuated degrees of freedom (DOF) to produce bi-directional

bending in combination with handle rotation and translation. Three tip DOF can be used to position the imager. The extra DOF can then be used to aim the imaging direction. We have determined closed form solutions for forward and inverse kinematics to enable position control and 1- DOF orientation control of the catheter tip. The kinematic algorithms were validated with a robotic test bed. The combination of positioning with imager rotation enables a wide range of visualization capabilities for improving the efficiency and effectiveness of intracardiac catheter interventions.

4.5 Nonlinear Information Fusion for Snakelike-Robot Backbone-Frame Localization

Authors

Gregory Chirikjian and Iulian Iordachita

Affiliations

Department of Mechanical Engineering and Laboratory for Computational Sensing and Robotics, Johns Hopkins University, Baltimore, MD, USA.

Abstract

Flexible snakelike devices (active catheters, steerable needles, cable-actuated elastic rods, flexible rubber hydraulic/pneumatic tubes, and concentric-tube robots) have the potential to be used in numerous applications in minimally invasive medical procedures. Regardless of the details of the particular morphology or architecture, a common problem arises in all of these cases. Namely, how can the best estimate of the shape and tool tip pose be found based on: (1) a prior mechanical model of the device and its interaction with tissues; and (2) posterior measurements of the shape of the device from sensor readings. In this presentation we develop methods for fusing these two sources of information for obtaining the most accurate estimate of the backbone curve and associated reference frames along the backbone. These methods do not assume linear (or linearized) models of the prior and posterior measurements, and therefore have the potential to more accurately capture the kinematics of snakelike devices than when using classical filtering theory.

4.6 Bifurcated Paths for Steerable Flexible Needles

Authors

Jaeyeon Lee and Wooram Park

Affiliations

University of Texas, Dallas, TX, USA.

Abstract

In this paper, we summarize our recent effort to reduce the tissue damage made by steerable flexible needles. We developed an algorithm to find a shortest path for a flexible needle that reaches multiple locations from a single entry point (i.e. port). The method was developed based on the observation that multiple locations can be reached by a flexible needle through insertion, partial retraction, rotation, and re-insertion of the needle. The resulting path was a bifurcated or branched trajectory. We showed that in 2D and 3D space this problem can be solved using geometric relationship between multiple tangent circles and spheres. Specifically we can find a needle insertion point, a corresponding insertion direction and lengths for insertion and retraction with which we can generate the optimal needle trajectory that

reaches two or three targets with the minimum tissue damage. To experimentally verify the method, we built a prototype of a needle insertion system, developed C# -based software to compute the optimal needle paths and performed the planned insertion using an open-loop controller.

5 Short Papers

A Novel Underactuated Wire-driven Flexible Robotic Arm with Controllable Bending Section Length

Zheng Li, *Member, IEEE, ASME,* Haoyong Yu, and Hongliang Ren

***Abstract*— This paper presents a novel underactuated wire-driven flexible robotic arm with controllable bending section length. It comprises an underactuated wire-driven flexible robotic arm, a rigid translating tube and the tube holder. The bending motion of the robotic arm is constrained by the tube. By translating the tube, the bending section length is controlled. With this design, the end-effector is able to reach more positions, or the workspace is increased. This is beneficial for surgical interventions or other operations in confined space.**

I. Introduction

Flexible manipulators such as tendon/cable/wire-driven robotic arms or concentric tubes are often used in surgical intervention. Compared to traditional ones, such as the Da Vinci robotic arm (a rigid shaft with a gripper at the distal end), the flexible manipulators provide more dexterity and enter the body with less trauma. The flexible manipulator is inserted into the human body, and can bend actively by tendon/cable/wire or precurved tubes. Examples are the distal dexterous unit (DDU) designed by Nabil Simaan, et al. [1], the wire-driven robotic arm designed by Zheng Li and Ruxu Du [2], the concentric tube robot designed by Pierre E. Dupont, et al. [3], the active cannulas designed by Robot Webster III, et al. [4]. In these designs, the manipulator backbone is either serpentine or continuum. The actuator number is much less than the number of structure degree of freedom (DOF), or the manipulator is underactuated. Except these designs, fully actuated design also exists, such as the double-screw-drive flexible manipulator proposed by Masakatsu G. Fujie's group [5].

For the underactuated flexible manipulators (UFM), the basic motion is bending. The distal end or the end effector is positioned and oriented by the backbone deformation. Existing tendon/cable/wire driven UFMs have one or more bending sections, and the length of each section is fixed. As a result, the distal end trajectory is fixed for a given bending section. This limits the workspace of the distal end. Although, the workspace can be extended by translating the UFM base, the orientation at the desired position is fixed. For concentric tube designs, the distal end trajectory is determined by the tube pre-curvature and tube interaction. Once the tube is fabricated, the distal tube curvature cannot be controlled. This also limits the distal end workspace and dexterity.

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Zheng Li is with the Singapore Institute of Neurotechnology, department of Biomedical engineering, National University of Singapore, Singapore. (Email: bielizh@nus.edu.sg)

Haoyong Yu is with the Singapore Institute of Neurotechnology, department of biomedical engineering, National University of Singapore, Singapore. (Phone: +65-66011590; Fax: +65-68723069; Email: bieyhy@nus.edu.sg)

Hongliang Ren is with the department of biomedical engineering, National University of Singapore, Singapore. (Email: ren@nus.edu.sg)

In this paper, a novel wire-driven UFM with controllable bending section is introduced. It comprises an underactuated wire-driven flexible robotic arm, a rigid tube, and the tube holder. Compared to existing wire-driven UFMs, the distal end is more dexterous, while compared to concentric tube robots the distal section curvature is controllable. The rest of the paper is organized as follows: section 2 presents the robot design; section 3 shows the kinematics and workspace analysis; and section 4 concludes the paper.

II. Robotic Arm Design

The following figure shows the robotic arm design. The robotic arm includes an underactuated wire-driven flexible section, a rigid tube, and the tube holder. The wire-driven flexible section is similar to our previous design [2]. It has several identical vertebras, and an elastic tube. Adjacent vertebras form a spherical joint, and the joint rotation follows the elastic tube bending. Four wires pass through all the vertebras and are attached to the last vertebra. These wires are grouped to two pairs and are orthogonally arranged. One wire pair controls the bending about X axis and the other wire pair controls the bending about Y axis. The robotic arm bending is planar. The bending angle and bending direction are controlled by the motion of the four wires. The rigid tube translates along the wire-driven flexible section. Vertebras within the tube are constrained and vertebras out of the tube are free of bending. Thus, the last constrained vertebra serves the base of the bending section. A gripper can also be placed at the distal end and is controlled by a wire.

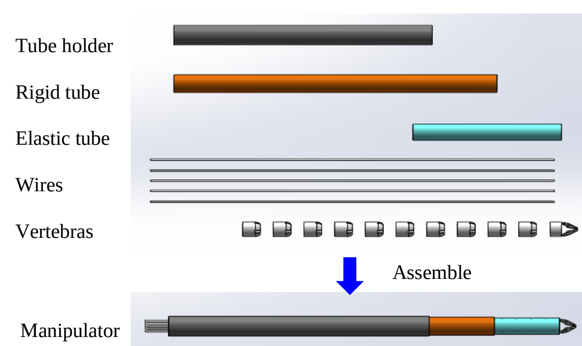


Figure 1. Underactuated wire-driven flexible robotic arm design

III. Kinematics and Workspace Analysis

The kinematics model is extended from the previous non-constrained wire-driven flexible robotic arm kinematics model. In this design, a translating tube is included, the bending motion is shown in Figure 2.

As in the figure, the world coordinate frame is set at the first joint rotation center. The arm rests on the Z_w axis. A local frame O_t-$X_tY_tZ_t$ parallel to O_w-$X_wY_wZ_w$ is located at the first unconstrained joint rotation center.

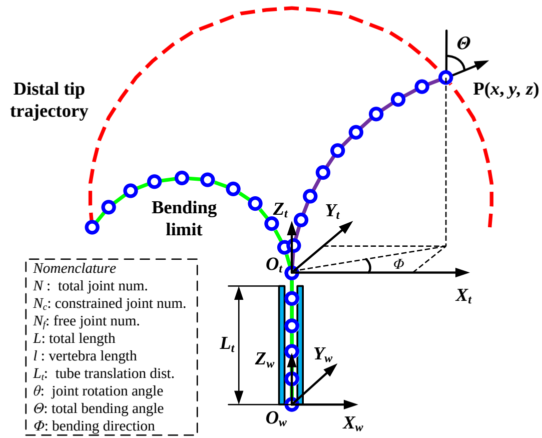


Figure 2. Robotic arm bending illustration

When the tube translates L_t along the flexible arm and the bending section bend Θ in the direction Φ, the distal end position $P(x,y,z)$ in the local frame can be derived as in the unconstrained case under the constant curvature assumption. The position in world coordinate frame is represented as following:

$$\begin{cases} x = \tilde{R}\sin\left[\left(N_f+1\right)\theta/2\right]\cos\left(\Phi\right) \\ y = \tilde{R}\sin\left[\left(N_f+1\right)\theta/2\right]\sin\left(\Phi\right) \\ z = N_c l + \tilde{R}\cos\left[\left(N_f+1\right)\theta/2\right] \end{cases} \tag{1}$$

$$\tilde{R} = l\frac{\sin\left(N_c\theta/2\right)}{\sin\left(\theta/2\right)} \tag{2}$$

where, l is the vertebra length, $N_c=[L_t/l+1]$ is the number of constrained vertebras, $N_f=N-N_c$ is the number of unconstrained vertebras, and θ is the joint rotation angle.

The robotic arm bending direction and joint bending angle could also be solved from the distal end position:

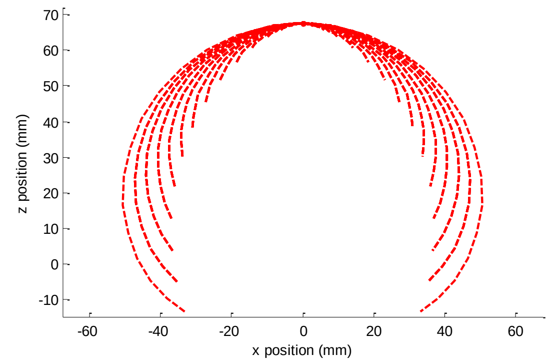
$$\Phi = \arctan\left(\frac{y}{x}\right) \tag{3}$$

$$\theta = \frac{2}{N_f+1}\arctan\frac{\sqrt{x^2+y^2}}{z-N_c l} \tag{4}$$

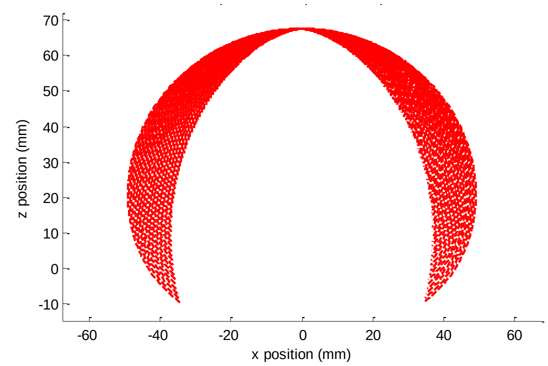
The workspace is derived from the kinematic model:

$$x^2+y^2+\left(z-N_c l\right)^2 = \tilde{R}^2 \tag{5}$$

It is noted that, the workspace contains N spheroidal surfaces as $N_c \in \{0,1,\ldots,N\text{-}1\}$. When the vertebra length reduces to zero, i.e. the robotic arm structure turns from serpentine to continuum, the workspace transforms to a solid volume. The following figure shows the cross-section view of the workspace of a serpentine robotic arm (N=15, $|\theta|\leq 14°$) and a continuum robotic arm with the same total length and maximum bending angle. The complete workspace can be obtained by spinning the cross-section view about Z_w axis. Compared to the workspace with fixed bending section, the workspace in this design is expanded a lot. The workspace can be further expanded by translating the tube holder.



(a) Cross-section view of serpentine robotic arm workspace



(b) Cross-section view of continuum robotic arm workspace

Figure 3. Wire-driven robotic arm workspace

IV. CONCLUSION

This paper presents the design and kinematic analysis of a novel underactuated wire-driven flexible robotic arm with controllable bending section length. Simulation results show that, compared to fixed-length bending section, the proposed design has expanded workspace. The expanded workspace of a serpentine UFM is a stack of spheroidal surfaces, while the expanded workspace of the continuum UFM is a symmetric spheroidal volume.

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Teleoperated Robotic Needle Steering with Cartesian Space Control

Ann Majewicz and Allison M. Okamura

***Abstract*— Asymmetric-tipped, robotically controlled steerable needles have the potential to improve clinical outcomes for many needle-based procedures by allowing the needle to curve and change direction within biological tissue. These needles are nonholonomic systems, which can be difficult to control manually using joint-space inputs of insertion and rotation at the needle base due to unintuitive kinematic constraints. A new method of controlling steerable needles through teleoperation of the needle tip in Cartesian space has been shown, in simulation, to result in significantly less targeting error and time to reach the intended target. In this work, we describe methods to implement Cartesian space teleoperation with a robotic needle steering system and present preliminary evaluation results.**

I. Introduction

Robotically steered needles show great promise for improving needle-based medical procedures (e.g. biopsy), as shown by prior demonstrations of needle steering applications in biological tissue [1], [2], [3]. Our needle steering technique exploits asymmetric tip forces to steer needles along constant-curvature paths during insertion [4]. Through a combination of needle insertion and axial rotation, the robotic needle can be controlled to follow a variety of paths, and reach a variety of targets, in three-dimensional space.

How to plan paths for steerable needles and control the needle along those paths is an active area of research [5], [6], [7], [8]. While autonomous image-guided needle steering systems have been developed, maintaining physician-in-the-loop guidance of steerable needles is desirable due to unpredictable needle behavior in biological tissue. Prior work has explored several teleoperation mappings to enable human-in-the-loop control of steerable needles [9], [10]; however, a Cartesian space teleoperation mapping has been shown to lead to more accurate needle insertions, with straighter paths, and less insertion time [11].

To implement Cartesian space teleoperation with a robotic needle steering system (Fig. 1(a)), it is important to both control needle curvature, and obtain pose measurements of the needle tip. Needle curvature can be controlled through duty-cycled spinning algorithms [12], [13]. During duty-cycled spinning, the needle path varies between a maximally curved path (resulting from needle insertion with no axial rotation) and a minimally curved path (resulting from needle insertion with continuous axial rotation, ideally zero curvature). Traditional duty-cycled spinning algorithms require continuous needle spin, which prevents the use of wired sensors such as force-torque sensors and electromagnetic (EM) trackers due to cable wind-up. As pose information of the needle tip is important for implementation of Cartesian space teleoperation, we have developed additional methods for duty-cycled control of needle curvature that do not require continuous needle rotation [14]. In this paper, we briefly describe the implementation choices and methods for our Cartesian space teleoperated robotic needle steering system, and present preliminary experimental results with a human operator.

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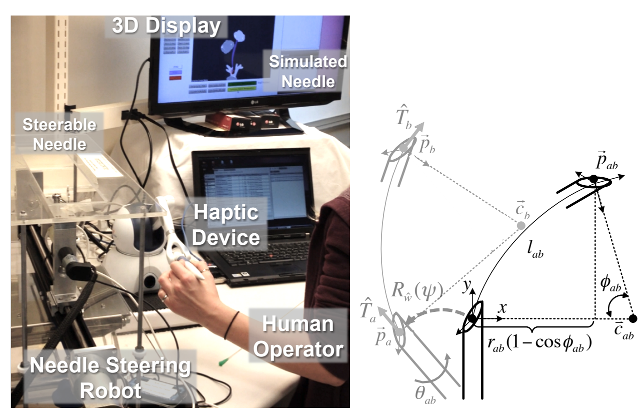
A. Majewicz and A. M. Okamura are with the Department of Mechanical Engineering, Stanford University, Stanford, CA 94305, USA, email: Ann.Majewicz@stanford.edu

II. Cartesian Space Teleoperation Algorithm

In traditional teleoperation of holonomic robots, the goal is often for the robot to follow a straight line path from its current to the desired position. For nonholonomic steerable needles, the needle must follow a constant-curvature arc to reach the desired pose due to kinematic constraints.

Our Cartesian space teleoperation algorithm iteratively generates constant-curvature local path segments for the needle based on the position error vector between the desired and current needle tip position (Fig. 1(b)). In practice, teleoperation is achieved by obtaining the desired needle tip position from a master haptic device, measuring the actual needle tip position and orientation with a medical imager or EM tracker, and implementing a control law that causes the needle tip to follow the curved path segment. Force feedback can be provided to the user to haptically render the kinematic constraints of the needle, and prevent the user from commanding unreachable needle paths [11].



(a) Teleoperated Needle Steering Robot (b) Local Path Geometry

Fig. 1: Teleoperated robotic needle steering system with Cartesian space control. Teleoperation is enabled by planning local constant curvature arcs between current needle pose and the desired needle tip position [11], [14].

III. Robotic Implementation

Our teleoperation system consists of a needle steering robot to actuate a steerable needle (e.g. [1]) and a master haptic device (i.e. Geomagic Touch, 3D Systems, USA) to allow for user control. We use a Cartesian space teleoperation algorithm where the user controls the desired location of the steerable needle tip [11] and a custom EM tracked steerable needle to measure the actual needle tip position [14].

A. Duty-Cycled Flipping for Needle Curvature Control

We developed a novel duty-cycled flipping algorithm that does not require continuous needle rotation, to control path curvature and enable use of wired sensors. In this algorithm, the needle inserts in discrete steps while flipping the needle bevel 180° in alternate directions (i.e., bevel left and bevel right) between each insertion to achieve a variable curvature path. The duty cycle, D, for this flipping method can be defined in terms of the time the needle inserts in its flipped orientation, T_{flipped} to its unflipped orientation, $T_{\text{unflipped}}$. The resulting curvature of the needle path, κ, is a function of the duty cycle and the maximum curvature of the needle.

$$D = 1 - \frac{T_{\text{flipped}} - T_{\text{unflipped}}}{T_{\text{flipped}} + T_{\text{unflipped}}}, \qquad \kappa = \kappa_{\max}(1 - D) \qquad (1)$$

B. Immersive Augmented Reality Environment

For an immersive augmented reality environment, we display desired targets to the user, as well as obstacles, in a three-dimensional virtual scene. This information could be obtained from patient-specific volumetric data from CT or MRI images. The desired needle path, a reconstructed needle path based on encoder readings from the robot and a simplified version of the needle kinematic model [4], and the real-time position tip of the steerable needle, measured through an electromagnetic tracker [14], are also displayed.

IV. Preliminary Experimental Evaluation

A human user teleoperated the steerable needle to a desired target, while avoiding two obstacles that prevented a straight-line path to the target. The user trajectory, desired needle path, reconstructed needle (from encoder readings), and measurements of the steerable needle tip using electromagnetic tracking are shown in Fig. 2. Figure **??** shows the distance of these measurements from the straight line path.

V. Conclusions

We have demonstrated teleoperated needle steering using a novel Cartesian space control teleoperation algorithm. Further work is needed to demonstrate the full range of capability of the teleoperated robotic needle steering system and improve transparency.

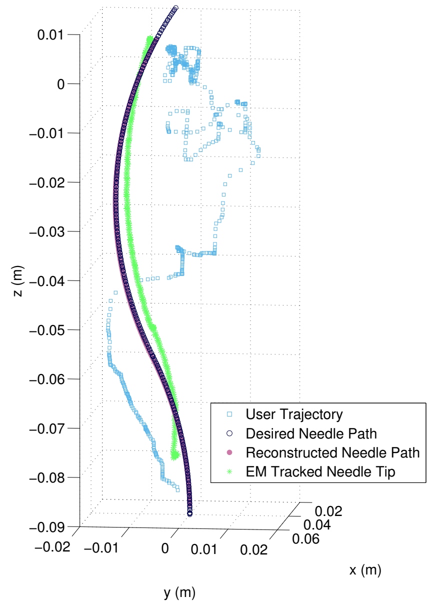


Fig. 2: Teleoperated steerable needle data including the user trajectory, desired needle path, reconstructed needle, and EM measurements of the steerable needle tip.

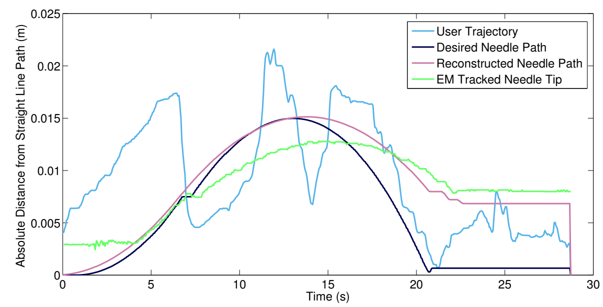


Fig. 3: Distance from a straight-line path to the target for the user trajectory, desired needle path, reconstructed needle, and EM measurements of the steerable needle tip.

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Model-less control of a flexible robotic catheter

Michael C. Yip, Paul. J. Wang and David B. Camarillo, *Member, IEEE*

***Abstract*—Flexible robotics are capable of navigating through unstructured environments such as a human body, where it can safely conform around environmental obstacles and constraints due to its compliant body. Because these environmental disturbances can affect the configuration of the robot in an unknown and unpredictable manner, conventional kinematics- and mechanics-based control methods can be inaccurate and can lead to robot singularities and controller instabilities. Using *model-less control*, we demonstrate control of a flexible robot manipulator without requiring a model. Furthermore, we show its application in a tissue ablation task using a flexible catheter. Model-less control provides a robust control strategy for flexible manipulators in unstructured environments, and offers a simple and effective alternative for controlling robots with complex kinematics and mechanics.**

I. Introduction

The major benefit of flexible robotic manipulators is that they can utilize their compliant bodies to navigate environments in a safe manner. Constraints and obstacles in the workspace cause the manipulators to conform into irregular configurations. Traditional closed-loop control techniques require a model (e.g. kinematics, mechanics) to estimate the configuration of the robot during each control cycle [1]. However, for a flexible manipulator, modeling the conformed configurations of its body can be extremely difficult, and current models tend only to be accurate for structured environments, and do not address interactions between the manipulator and unknown and unstructured environments. This leads to a diminished reachable workspace and possible instability issues [2].

Because of the complexity of the kinematics and mechanics of these manipulators, including the fact that they often exhibit over- or under-actuation (n actuators, m degrees of freedom, $n \neq m$), an alternative method is presented. We demonstrate that closed-loop task-space control can be achieved without using a model using *model-less control*, providing a simple and efficient way to control complex robot manipulators, and furthermore overcoming limitations of model-based systems.

II. Methods

A. The model-less control framework

In this paper, the model-less controller [2] is defined for a tendon-driven continuum manipulator (Figure 1). Model-less control comprises a Jacobian estimation method that is applied to a task-space closed-loop controller, described below.

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M.C. Yip and D.B. Camarillo are with the Department of Bioengineering at Stanford University, Stanford, CA, 94305 USA. (e-mail: mcyip@stanford.edu, dcamarillo@stanford.edu)

P.J. Wang is with the School of Medicine (Cardiovascular Medicine) and with the Department of Bioengineering at Stanford University, Stanford, Ca 94305 USA. (email: paul.j.wang@stanford.edu)

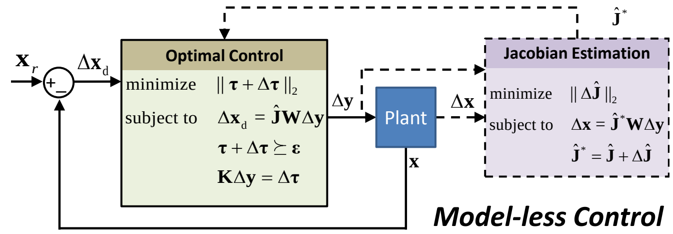

Fig. 1: Flowchart showing the closed-loop control for a tendon-driven continuum manipulator (solid lines) and the constrained optimization method for model-less control (dashed lines). $\mathbf{x}$ is the manipulator position, $\mathbf{y}$ is the actuator positions, τ is the cable tensions, $\mathbf{K}$ is a stiffness mapping, and $\hat{\mathbf{J}}$ is the robot Jacobian estimate.

Jacobian estimation: At the heart of the model-less control method is the Jacobian estimation method (dashed lines in Figure 1). It is defined as a constrained optimization method that finds an estimate of the Jacobian, $\hat{\mathbf{J}}$, that satisfies the mapping $\Delta\mathbf{x} = \hat{\mathbf{J}}\mathbf{W}\Delta\mathbf{y}$. This is always an under-constrained problem; therefore, a least-norm solution is chosen such that the change in $\hat{\mathbf{J}}$ is minimized at every estimation step, resulting in smooth changes in the Jacobian estimate, shown below:

$$\begin{aligned} \underset{\hat{\mathbf{J}}^*}{\text{minimize}} \quad & ||\Delta\hat{\mathbf{J}}||_2 \\ \text{subject to} \quad & \Delta\mathbf{x} = \hat{\mathbf{J}}^*\mathbf{W}\Delta\mathbf{y} \\ & \hat{\mathbf{J}}^* = \hat{\mathbf{J}} + \Delta\hat{\mathbf{J}} \end{aligned} \quad (1)$$

where the new Jacobian estimate is $\hat{\mathbf{J}}^*$. It should be noted that more constraints can be added to the optimization method to further constrain the solution space and potentially improve the Jacobian estimate (e.g. geometric constraints).

Optimal control: The Jacobian estimate is used in a task-space closed-loop method (shown in solid lines in Figure 1). For a continuum manipulator driven by co-activated tendons, this is an over-constrained system and therefore we again use constrained optimization to solve for actuator displacements $\Delta\mathbf{y}$. In this solution, the controller minimizes tendon tensions τ and therefore reduces internal manipulator loading.

B. Experimental Setup

The model-less controller was implemented on a continuum manipulator, comprising a flexible backbone (280 mm length) and two 0.6 mm steel tendons. Tension sensors measure cable tensions during tendon actuation, and an insertion motor drives the robot forwards and backwards. A 50 Hz camera provides position tracking at a resolution of approximately 0.5 mm per pixel. In addition to closed-loop position tracking, a minimum tendon tension of $\tau = 0.3$ N was maintained using the constrained optimization method during each control cycle.

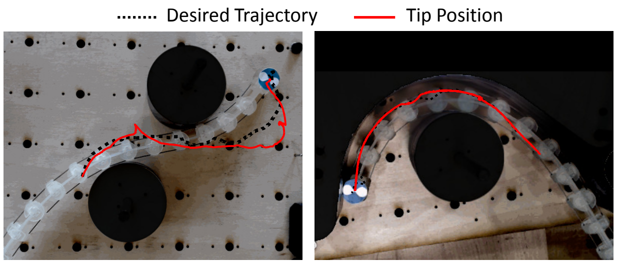

Fig. 2: A flexible manipulator tracks a trajectory without knowledge of environment. Tracking of the trajectory is maintained in spite of unknown obstacles that constrain and conform the manipulator into irregular configurations.

Since the problem is convex, optimization was performed in $\sim 10\mu$ sec using the CVXgen convex optimization solver [3].

III. Results

A. Movement in Obstacles

Figure 2 shows the model-less controller navigating two environments with obstacles. The first environment involves navigating two posts that results in an s-shaped conformation of the robot. In the second case, the model-less control moves the robot through a channel resulting in an approximately $135°$ turn. In both cases, model-less control is effective in following the trajectory despite the significant environmental effects at unknown locations along the manipulator body. The channel constraint presents a case where one actuation input (insertion) becomes inverted as the robot moves through the constraint; without explicit knowledge of the environment, model-based methods would not be able to account for the inversion and would therefore result in a positive feedback loop and instability.

B. Tissue contact simulation

One promising medical application for flexible robots is catheter ablation. Applying sufficient pressure to soft tissue and performing ablation allows for targeted necrosis of cells that cause cardiac arrhythmia. Contiguous linear ablations segregate healthy tissues from erroneous and arrhythmia-inducing electrical signals. Control of these contiguous paths are most effectively done in task-space and therefore is a good environment for applying the proposed model-less controller.

Figure 3 shows the continuum manipulator applying pressure and tracing a linear path on different silicone tissue phantoms. For various stiffnesses, the model-less controller is able to trace a user-defined desired trajectory while applying pressure to the environment. Because the forces are not being explicitly controlled, The indentation of the tissue and the compliance of the manipulator dictate the forces applied to the environment, and this is also reflected in the cable tensions. Rise in cable tensions and force reflect the controller's attempt to reduce the y-axis position error. The compliance of the tissue environment and the compliance of the manipulator together keep interaction forces to a safe level.

[1] Elastic stiffness for in-vivo heart tissue fall between $0.0249 - 0.136$ MPa (healthy to congenital cardiomyopathy) [4].

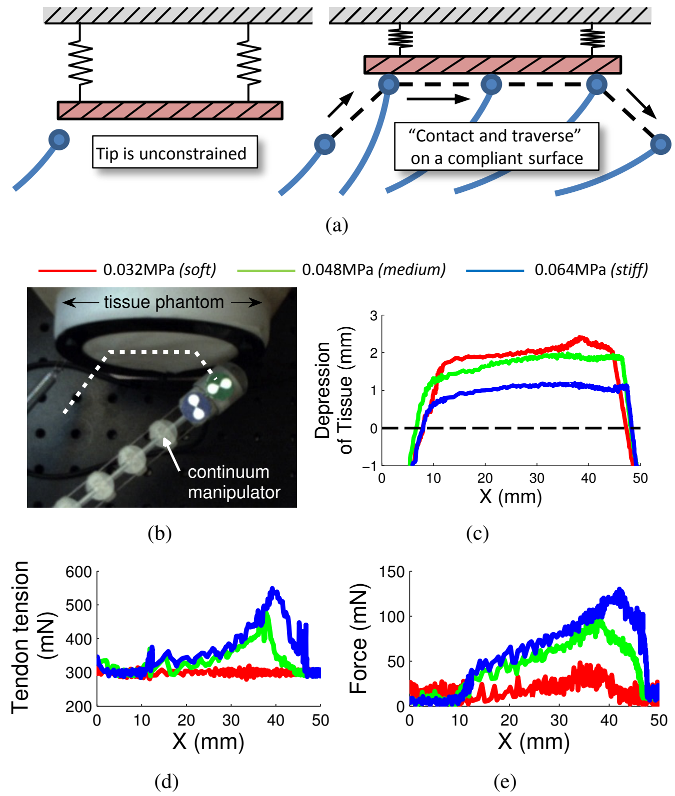

Fig. 3: A robot manipulator coming into tip-contact with soft constraint and tracing a path along the constraint (a), as is done for cardiac tissue ablation. (b) shows the environmental setup. For a given trajectory, the tracking accuracy (c), tendon tensions (d) and interaction forces (e) vary for different tissue elastic stiffnesses $(0.32, 0.48, 0.64$ MPa$)$[1].

IV. Discussion

Model-less control provides a valuable approach to controlling a robot manipulator in the following situations: (1) when the robot has complex kinematics or mechanics that are difficult to model, and (2) when there are unknown disturbances (e.g. environmental constraints) that affect the manipulator in an unpredictable way. It can be considered a local learning method that identifies the approximate mapping between actuator displacements and actuator outputs while satisfying any number of user-defined constraints. The compliance of a flexible manipulator can then be used advantageously in constrained environments without concerning the user with modeling its complex bending properties. Future work will involve investigating force regulation as a control constraint.

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Automated Pointing of Cardiac Ultrasound Catheters

Paul M. Loschak, Laura J. Brattain, Robert D. Howe
Harvard School of Engineering and Applied Sciences, Cambridge, MA, USA

***Abstract*—Automatically positioning cardiac imaging catheters within the heart can improve diagnoses and treatments of medical conditions such as cardiac arrhythmias, including atrial fibrillation. Imaging catheters are unique compared to many position-controlled flexible instruments because device orientation must be specified in order to point the imager at the target. For example, ultrasound imaging (intracardiac echo, or ICE) catheters are steered by four actuated degrees of freedom (DOF) to produce bi-directional bending in combination with handle rotation and translation. Three tip DOF can be used to position the imager. The extra DOF can then be used to aim the imaging direction. We have determined closed form solutions for forward and inverse kinematics to enable position control and 1-DOF orientation control of the catheter tip. The kinematic algorithms were validated with a robotic test bed. The combination of positioning with imager rotation enables a wide range of visualization capabilities for improving the efficiency and effectiveness of intracardiac catheter interventions.**

I. Introduction

Ultrasound imaging catheters have been routinely used in cardiac catheter procedures for over a decade [1]. These catheters are inserted into the patient's vasculature (e.g. femoral vein) and manually navigated to the heart, where a phased-array transducer within the catheter tip acquires B-mode images of cardiac structures. Compared with external probes, ultrasound (US) catheters can achieve higher quality views of targets in the near-field with higher acoustic frequencies, reducing aberration and attenuation due to layers of intervening muscle, fat, and other tissues. Controlling US imaging catheters requires the clinician to maneuver the imaging plane by manually turning control knobs and rotating and advancing the catheter handle while the shaft of the catheter winds through tortuous blood vessels. Unfortunately, this makes it highly challenging to align the image plane with a target. Moving between targets can require extensive time and skill to obtain an adequate view. This has largely limited the use of catheter US to a few critical tasks such as transeptal puncture [2].

II. Methods

The flexible instrument bending model is a set of closed-form forward and inverse kinematic solutions derived from geometric analysis and robot kinematics [3]. This is the first model known to the authors capable of calculating both the position and orientation of the catheter tip for catheters with two bending directions. Orientation information enables calculation of the ICE image direction. The model assumes that catheter bending occurs in a bending plane (neglecting the effects of torsion). We also assume that the catheter bends with a constant radius of curvature, and that dynamic effects

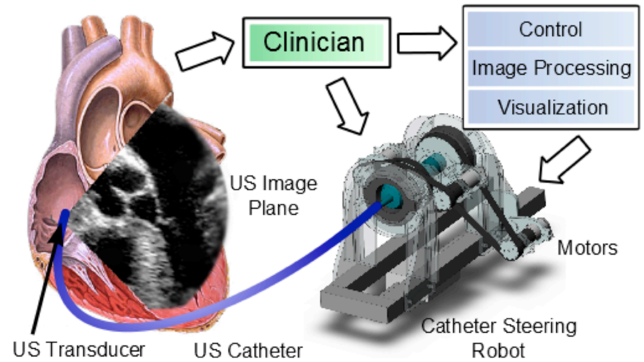

Fig. 1. System diagram

of catheter motion are negligible. Many flexible manipulators exhibit positional joint coupling in bi-directional bending. The bi-directional bending catheter is a manipulator in which pitch and yaw can occur simultaneously. It is assumed that applying pitch then yaw will yield the same kinematic results as applying yaw then pitch. The orientation of the US imager can therefore be determined by rotating the mobile coordinate frame about an equivalent axis [4]. Previous work validated this claim [3].

A robot actuates the catheter handle knobs by timing belts driven by brushed DC motors. A low-level control loop (digital positioning controllers) drives each actuator to reach the commanded joint angles. The closed loop controller relies on sensor readings from electromagnetic (EM) trackers mounted to the distal tip of the ICE catheter.

III. Experiments

The system is capable of manipulating an ICE catheter and its imaging plane in several specific motions.

A. Position Control

Accurate position control of the catheter tip is achieved by coordinating pitch, yaw, and translation. Inverse kinematic calculations (Fig. 2) are used for large navigation motions and an inverse Jacobian method is used to achieve small position adjustments. Fig. 6 shows the ICE catheter tracing 6 cm square trajectories in orthogonal planes, with average 1.9 mm RMS error.

B. Image Scanning

Coordinated motion of the pitch, yaw, and roll knobs can rotate the angle of the US image without displacing the ICE catheter (Fig. 3). This is useful for imaging a large area of the heart while maintaining the ICE catheter tip in a safe position. Previous work has demonstrated the utility of reconstructing 2D image slices into a 3D or 4D (3D + time) volume [5, 6].

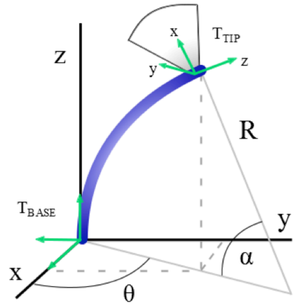

Fig. 2. Catheter bending kinematics

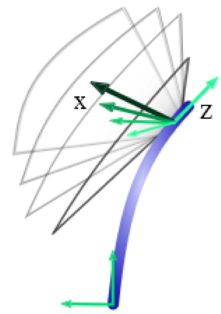

Fig. 3. Schematic of imaging plane sweeping

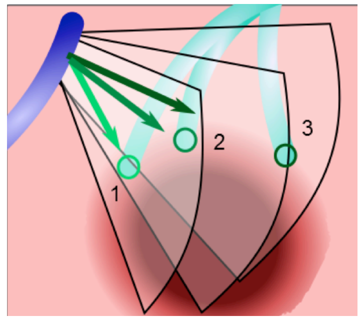

Fig. 4. Schematic of instrument tracking

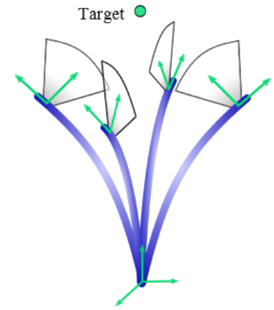

Fig. 5. Schematic of ICE circling and pointing at a target

The system is able to achieve accurate angular adjustments to within 0.25° RMS error while maintaining catheter tip displacement with 1.0 mm RMS error (Fig. 7). The resulting 3D image mosaics are superior to current clinical 3D ultrasound images.

C. *Instrument Tracking*

The system can track a working instrument (such as an ablation catheter) by image scanning to maintain continuous visualization of its tip (Fig. 4). An electromagnetic tracker in the instrument allows the system to continuously minimize the angular error between the plane of the imager and the location of the instrument tip. Visualizing the instrument tip allows clinicians to directly monitor instrument-tissue interactions. This is an important new capability for cardiac catheter interventions, which are presently guided by fluoroscopy with limited ability to visualize soft tissues. Experimental results show imager pointing with 0.25° RMS error and catheter tip movements with 1.9 mm RMS error (Fig. 8).

D. *3D Pointing*

This capability enables navigation around a target while imaging it from all sides (Fig. 5). The target tissue can be located anywhere within the workspace that the US imager can visualize. The ICE catheter is commanded to maintain a fixed distance from the target, and the solution set of achievable catheter locations is calculated. During testing the robot reached its commanded positions with 1.6 mm RMS error and pointed the imager at the object with 0.17° RMS error (Fig. 9).

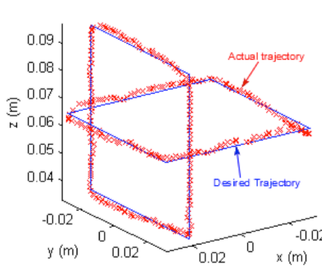

Fig. 6. Catheter position control experimental results

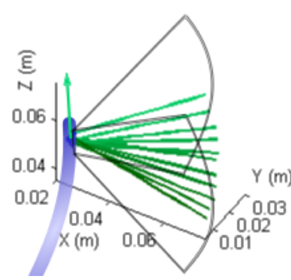

Fig. 7. Sweeping test results. Green vectors represent the image plane pointing at the target.

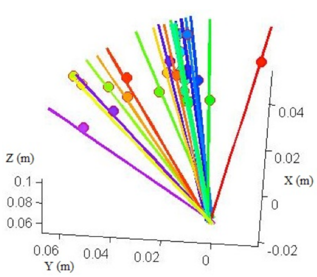

Fig. 8. Instrument tracking results. Dots represent target instrument positions, vectors are image plane centerlines.

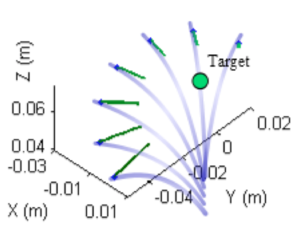

Fig. 9. 3D pointing results for automatically aiming the image plane (green vectors) at a target.

IV. Conclusions

Controlling the position and orientation of the ICE catheter enables accurate pointing of the imaging plane. This allows, for the first time, large-area intracardiac 3D imaging, as well as continuous monitoring of the interactions between an instrument and the tissue target. The tests described in this study demonstrate the utility of ICE catheter position and orientation kinematics together to robotically enhance visualization.. This will enable clinicians to move ICE to a safe location and image structures that are difficult to locate by manual manipulation. Future work aims to examine system inaccuracies, safety boundaries inside the heart, and prepare for *in vivo* studies. Robotic control of ICE promises to shorten procedure times, improve patient outcomes, and reduce the training time required to master ICE.

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Nonlinear Information Fusion for Snakelike-Robot Backbone-Frame Localization

Gregory Chirikjian and Iulian Iordachita [‡]

Abstract

Flexible snakelike devices (active catheters, steerable needles, cable-actuated elastic rods, flexible rubber hydraulic/pneumatic tubes, and concentric-tube robots) have the potential to be used in numerous applications in minimally invasive medical procedures. Regardless of the details of the particular morphology or architecture, a common problem arises in all of these cases. Namely, how can the best estimate of the shape and tool tip pose be found based on: (1) a prior mechanical model of the device and its interaction with tissues; and (2) posterior measurements of the shape of the device from sensor readings. In this presentation we develop methods for fusing these two sources of information for obtaining the most accurate estimate of the backbone curve and associated reference frames along the backbone. These methods do not assume linear (or linearized) models of the prior and posterior measurements, and therefore have the potential to more accurately capture the kinematics of snakelike devices than when using classical filtering theory.

1. INTRODUCTION

Though snakelike robots have arisen in a variety of independent efforts since at least the late 1960s, the use of continuum (backbone curve) models to capture the shape of arbitrary snakelike robots with either continuous or discrete morphologies began in ernest in the early 1990s (see [1–3] for details). Since then, a number of variations on the theme of snakelike devices have appeared in the medical device literature, including concentric tube robots [4,5], tendon-actuated elastic rods [6,7], and flexible steerable needles (including unicycle [8] and bicycle [9] models) and their application and generalization [10]. In parallel with these developments, and largely independently from them, the field of "continuum robots" has evolving over the past decade. One of the first efforts along these lines was the flexible microactuators in [12], and more recently elephant trunk robots [13] have been studied.

An important issue that arises in the application of any of these devices to real-world problems is the fusion of information about the shape of these robots from different sources. One source of information is the inputs provided to actuators that are then converted to form a probabilistic prior through a mechanical model. For example, the lengths of tendons in a tendon-driven manipulator, or joint angles in a revolute manipulator, or the history of pushes and twists of a flexible needle can be mapped through the forward kinematics to obtain an initial estimate of what the shape should be. But real systems have noise, and so instead of generating the exact shape of the snakelike device, an ensemble of shapes will be generated. This concept has been used for flexible steerable needles in [8,14,15]. In many ways this is analogous to problems in mobile robot localization [16], with arclength along the backbone replacing time as the independent variable.

Previous efforts have developed either deterministic observers to incorporate measurements (e.g., [17]) or linear (or linearized) stochastic filtering models [11]. In this paper we develop nonlinear information-fusion methods for obtaining the best estimate of snakelike robot shape. Our method is based on the concept of $SE(3)$ Gaussian distribution $f(g;\mu,\Sigma) \doteq$

$$\frac{1}{(2\pi)^{d/2}|\Sigma|^{\frac{1}{2}}} \exp\left(-\frac{1}{2}[\log(\mu^{-1}g)^{\vee}]^{T}\,\Sigma^{-1}\log(\mu^{-1}g)^{\vee}\right) \tag{1}$$

where $d = 6$ and $g = (R,\mathbf{t}) \in SE(3)$ (the group of special Euclidean, or rigid-body, motions). If

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[†]The authors are with the Department of Mechanical Engineering and Laboratory for Computational Sensing and Robotics, Johns Hopkins University, Baltimore, MD, USA, {gregc,iordachita} at jhu.edu

$f(g;\mu_1(s),\Sigma_1(s))$ is a prior model that parameterizes the probability of pose of a reference frame attached to a snake robot at arclength s along its backbone, e.g., for the needle case by solving the sorts of Fokker-Planck equations in [8, 18], and if embedded in the snake robot is a fiber-optic curvature sensor that provides an alternative source for measuring the local curvature at discrete points along the backbone of the snake robot, then a second estimate of the pose distribution can be obtained, which will have a similar form, but different values of mean and covariance, resulting in $f(g;\mu_1(s),\Sigma_1(s))$ If the problem were in the linearized setting, the fusion of these pdfs (which is essentially their multiplication and renormalization) would be trivial, which corresponds to the update step in a Kalman filter. But for nonlinear measurements, the update step is quite involved, as described in [19], which provides algorithms for obtaining the best $(\mu_3(s),\Sigma_3(s))$ such that $f(g;\mu_3(s),\Sigma_3(s)) \approx$

$$\frac{f(g;\mu_1(s),\Sigma_1(s))f(g;\mu_2(s),\Sigma_2(s))}{\int_G f(h;\mu_1(s),\Sigma_1(s))f(h;\mu_2(s),\Sigma_2(s))dh}$$

The poster/presentation associated with this paper will explain the mathematical steps involved in performing this fusion in the context of snake robot models and illustrate initial hardware implementations. (Space is not available here to do so).

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Bifurcated paths for steerable flexible needles

Jaeyeon Lee and Wooram Park

Abstract— **In this paper, we summarize our recent effort to minimize the tissue damage made by flexible medical needles while the needle targets multiple locations. We developed an algorithm to find a shortest path for a flexible needle that reaches multiple locations from a single entry point (i.e. port). The method was developed based on the observation that multiple locations can be reached by a flexible needle through insertion, partial retraction, rotation, and re-insertion of the needle. The resulting path is a bifurcated or branched trajectory. We showed that in 2D and 3D space this problem can be solved using geometric relationship between multiple tangent circles. Specifically we can find a needle insertion point, a corresponding insertion direction and lengths for insertion and retraction with which we can generate the optimal needle trajectory that reaches two or three targets with the minimum tissue damage. To experimentally verify the method in the 2D environment, we built a prototype of a needle insertion system, developed C#-based software to compute the optimal needle paths and performed the planned insertion using an open-loop controller.**

I. INTRODUCTION

For recent years, many research activities for a flexible needle with a bevel tip have been reported. They include modeling of kinematics and mechanics, path planning, and control. Webster III et al. [1] have developed a nonholonomic kinematic model for insertion of the flexible needles with a bevel tip. This model was adopted in many works [2], [3], [4]. Alterovitz et al. [5] developed a 2D planner for insertion of the flexible needles with which the needle can reach a planar target through a minimized path with obstacle avoidance. Duindam et al. [6] used geometric inverse kinematics to generate a needle path. Park et al. [2] introduced the path-of-probability (POP) method in stochastic environment to obtain a needle path. Based on the unicycle model in [7], the needle trajectory can be assumed that it follows a circular arc when it is inserted into soft tissue without any rotation. In many works in this topic, either of two control inputs (i.e. pushing and rotating) is applied at a time for simple modeling and analysis, even though the simultaneous inputs can generate complicated trajectories. Therefore the resulting trajectory is a combination of multiple circular arcs which are tangent to each other. We report our recent effort to develop a method to generate path with which the needle can reach multiple locations from a single insertion position while the tissue damage is minimized. This can be applied to drug delivery and biopsy at multiple locations.

Jaeyeon Lee is with the Department of Electrical Engineering, University of Texas at Dallas, 800 W. Campbell Rd., Richardson, TX, USA.

W. Park is with the Department of Mechanical Engineering, University of Texas at Dallas, 800 W. Campbell Rd., Richardson, TX, USA.{jaeyeon.lee, wooram.park}@utdallas.edu

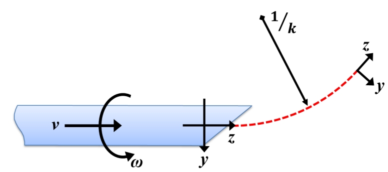

Fig. 1. Control inputs and a reference frame for a flexible needle [7].

II. TARGETING MULTIPLE LOCATIONS USING FLEXIBLE NEEDLES

A. Modeling for flexible needles

Control inputs and a reference frame for a flexible needle are shown in Fig. 1. Based on this, a non-holonomic and unicycle-based needle model for flexible needles was developed in [7]. In Fig. 1, κ is the needle curvature generated by the bevel tip and $v(t)$ and $\omega(t)$ are kinematic control inputs. $v(t)$ is the insertion velocity, and $\omega(t)$ is the rotational velocity around the needle axis. This model is an extension of the unicycle model to 3D space.

B. Targeting multiple locations

In medical applications of the flexible needle with a bevel tip, it is worth finding a method to steer the flexible needle to hit multiple targets. Drug injection or biopsy at multiple locations can benefit from this new approach. We are inspired by a simple test where the flexible needle is inserted, partially retracted, rotated by 180° and re-inserted. Through this manipulation the needle can reach two positions from a single insertion point on the plane. We can also target locations in 3D space using other rotation angles in the test. We proposed a method to find an optimal needle path that reaches multiple locations while minimizing tissue damage in 2D and 3D space [3], [4].

2D space

As in [3], the geometry for a two-target problem in needle insertion in 2D space is shown in Fig. 2. The needle is inserted at P_{in}, and then its tip will follow the left circle to the first target P_1. Next the needle is retracted to the turning point T_1. At this location, the needle is rotated around its axis by 180°, followed by the re-insertion to the second target P_2. Note that this insertion method can give multiple solutions in terms of the insertion point P_{in} as long as the insertion angle θ (see Fig. 2) is determined correctly.

The cost function which measures the length of the needle trajectory is defined as

$$C(x) = \ell_1 + \ell_2, \tag{1}$$

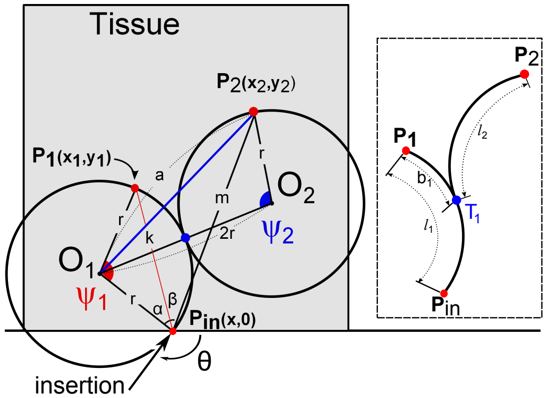

Fig. 2. The two tangent circular arcs show the needle trajectory that reaches two targets.

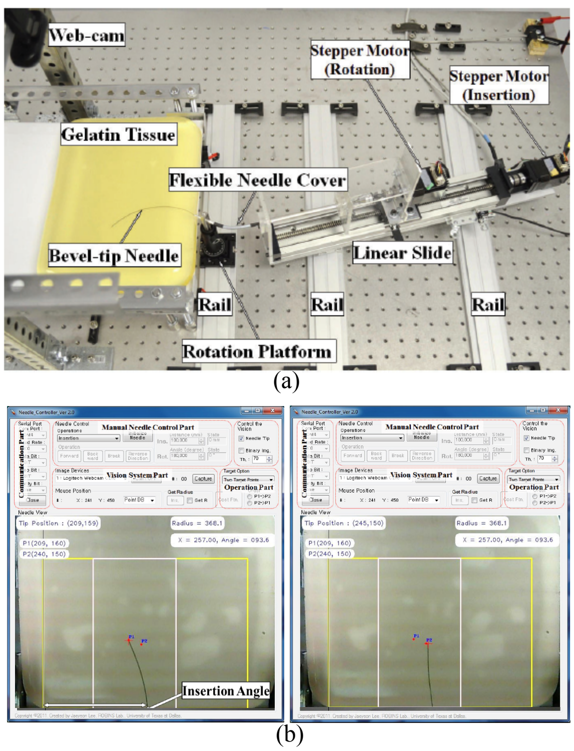

Fig. 3. (a) Hardware and (b) software for needle insertion.

where x is the location of the insertion point, ℓ_1 is the insertion length between the insertion position and the first target (P_1) along the circular arc around O_1 and ℓ_2 is the insertion length between a turning point (T_1) and the second target (P_2) along with the circular arc around O_2. The cost function which is a sum of all insertion lengths in forward direction can be viewed as tissue damage. By minimizing this cost function, we can select the optimal insertion location (P_{in}) and insertion direction (θ) so that the tissue damage is minimized as shown in Fig. 2. For the three-target problem in 2D space, a similar approach can be applied with three tangent circular arcs [3]. We verified the applicability of this approach using the actual insertion system and the control software shown in Fig. 3.

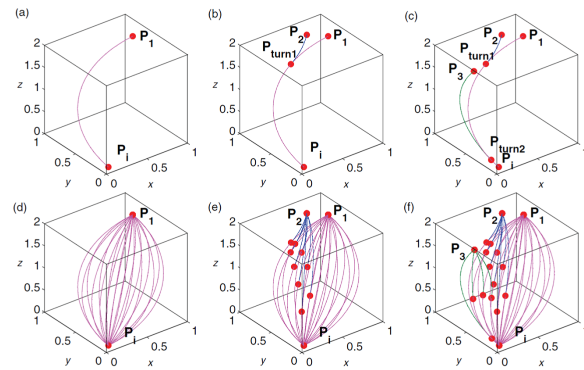

Fig. 4. A multiple-target problem in 3D space [4].

3D space

The method for the 2D problem can extended to the 3D space [4]. In this case the rotation angle in the needle manipulation is not limited to 180°. A suitable angle should be found in the algorithm such that the needle can reach the targets. The first row of Fig. 4 illustrates the needle trajectory targeting three locations in the 3D space. The insertion location, the insertion angle, the two turning points and the rotation angles are determined. These are found through the exhaustive search as illustrated in the second low of Fig. 4.

III. Conclusion and Future Work

In this paper, we introduced our recent research effort to minimize the tissue damage by flexible needles when we target multiple locations using flexible needles. A geometry-based approach was used to develop algorithms to generate the optimal insertion location and direction for needle manipulation in the 2D and 3D space. The 2D approach was verified experimentally, and the assessment for the 3D approach is planned as future work.

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